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ABSTRACT:

This paper offers an expressivist account of logical form, arguing that in order to fully understand it one must examine what valid arguments make us *do* (or: what Achilles does and the Tortoise doesn't, in Carroll's famed fable). It introduces Charles Peirce's distinction between symbols, indices and icons as three different kinds of signification whereby the sign picks out its object by learned convention, by unmediated indication, and by resemblance respectively. It is then argued that logical form is represented by the third, iconic, kind of sign. It is noted that icons uniquely enjoy partial identity between sign and object, and argued that this holds the key to Carroll's puzzle. Finally, from this examination of sign-types metaphysical morals are drawn: that the traditional foes metaphysical realism and conventionalism constitute a false dichotomy, and that reality contains intriguingly inference-binding structures.

Introduction

Much development in twentieth century analytic philosophy is arguably driven by working out the semantics underlying *truth*. A widely-held *representationalist* model of language holds that its primary purpose is to state "facts" in mind-independent reality, and thus the model naturally accompanies a certain metaphysical realism. It has shaped metaphysics, metaethics and numerous other philosophical areas to an extent hard to appreciate unless one steps outside representationalism – not an easy task.

It's worth noting that the criteria for whether propositions are fact-stating underwent significant evolution. For early logical positivists, all statements were divided into those with "literal significance" and those without, and the former were so by virtue of offering "empirical hypotheses"[1]. Thus "The cat is on the mat" is literally significant because a cat-on-mat *experience* might be had in relevant situations. However, crisp criteria for genuinely empirical hypotheses were harder to find than the logical positivists initially supposed. So Quine presented another, more 'purely semantic' criterion of factuality: if a literally significant discourse were regularized into a single theory in first-order logic, its bound variables would have values.[2] Now "The cat is on the mat" is factual because in $\exists x (Cx \ \& \ Oxm)$ suitably interpreted, x binds to George.

Meanwhile, various philosophers opposed this broad mainstream consensus with forms of antirealism which questioned whether if many important areas of human thought, e.g. ethics, were regularized into first order logic (if this were even possible), they would yield suitably denoting variables. Thus they opposed metaphysical realism with some form of *conventionalism* arguing that terms such as 'the Good', or 'God', rather than denoting existent objects, possess other entrenched, assertion-warranting social functions. Pragmatism, instrumentalism and much later-Wittgensteinian thought all fall under this heading. This dialectic between metaphysical realism and conventionalism concerning the denotation of philosophically contested terms runs deep through 20th century philosophy.

So much for truth. What semantics underlies *logical validity*? Reasoning from premises to conclusion seems somehow to consist in more than the component propositions' individual assertions of fact, insofar as the premises *justify* the conclusion. This issue famously troubled the very representationalist early Wittgenstein, leading him to declare logic transcendental[3]. This was not enthusiastically embraced by his successors, however, and recently new approaches to the 'foundations of logic' have emerged. These include a *model-theoretic account* which argues that a conclusion follows from premises if it is true in every model in which they are (effectively enfolding validity into representationalism by extending the latter to other possible worlds), and

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[1] A.J. Ayer, *Language, Truth and Logic* (London: Dover, 1953), p. 2.
[2] W.V.O. Quine, "On What There Is", *From a Logical Point of View* (Cambridge, MA: Harvard University Press, 1953), pp. 1-19.
[3] Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, trs. G.E.M. Anscombe (London: Routledge, 1961), section 6.13.

an *inferentialism* which argues that conclusion follows from premises if it may be derived from them by step-wise application of primitive inference-rules (where one might argue the essence of logical validity remains unreduced).[4]

This paper offers a different, *expressivist*, approach. *Contra* the model-theoretic account, it argues that logical form is not in any way denoted. *Contra* standard inferentialism, it argues that to fully understand logical form we need to unpack even for the simplest inferential forms what it makes us *do*.

Achilles and the Tortoise: 'The Hardness of the Logical Must'

  ARG) Socrates is a human being.   **PREMISE: P1**
           All human beings can be killed. **PREMISE: P2**
           Therefore Socrates can be killed. **CONCLUSION: C**

In an argument such as this most rational English speakers *see* a pattern which inclines them to *do* something, namely, if they believe or suppose **P1** and **P2**, to infer **C**. How does this work? What makes it happen? (And what do we mean by 'make'?) Lewis Carroll's justly famed fable of Achilles and the Tortoise[5] confronts these questions. It adroitly highlights both the existence and the puzzlement of a certain feature of our necessary reasoning, namely a bindingness on the *actions* of rational agents (specifically their inferencing) which appears *a-causal*, yet nonetheless intriguingly compelling.

The two mythical racers contemplate a valid argument, such as **ARG)**[6]. The Tortoise asks Achilles what he would say to someone who accepts the premises but not the conclusion. Achilles has surprising trouble with this. He writes down a conditional to express what he sees as manifestly true and missed by the Tortoise:

      COND1): If **P1** and **P2** are true, then **C** must be true.

The Tortoise then asks what difference this inscription makes to what he should *do* with respect to inferring **C**, even if he accepts **P1** and **P2**. Achilles resorts to a further written conditional:

      COND2): If **P1** and **P2** and **COND1** are true, then **C** must be true.

which fails to move the Tortoise, and so on through further conditionals towards infinity, with the impasse never resolved.

Achilles sees the Tortoise as recalcitrant as his slow companion refuses to be bound by a clearly valid argument. The Tortoise seems to understand the premises, yet inferring **C** is something he refuses to do. Here one glimpses an *internalism* about logic, whose analogy to metaethics is arguably no accident, whereby if one is not motivated to *act* by logical norms, it seems that one does not fully understand them. Yet *via* his recalcitrance the Tortoise usefully makes the binding process − normally invisible due to its ubiquity − visible.

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[4] For a large current research project in this area, see http://www.st-andrews.ac.uk/~arche/projects/logic/
[5] Lewis Carroll, "What the Tortoise Said to Achilles", Mind **104** (1995), pp. 691–3.
[6] In the original article Achilles and the Tortoise are actually examining Euclid's first geometrical proof, but this example has been substituted for ease of exposition.

We have seen that the disparity between Achilles and the Tortoise cannot be remedied by further explanatory signs, as the conditionals tend to infinity. I suggest that what is now exposed is *a structural isomorphism shared by a written sign* (**ARG)**) *and an act* (inferring **C** from **P1** and **P2**). This might seem a curious idea. Yet the fact that the further conditional statements are useless shows that what is seen by Achilles must be *already present in ARG) and the way he acts on it*. But how? I will argue: by means of a special kind of sign which current representationalism overlooks.

Iconic Signs: Not Symbolic, or Indexical

Peirce distinguished three kinds of sign by the way in which the sign picks out its object, and thus gains meaning.[7] *Symbols* gain meaning by some arbitrary habit or convention which must be learned (e.g. "banana" in English). All words are symbolic to some degree. However as Perry showed[8], language also includes signs which pick out objects by some direct 'indicating' or 'pointing' relationship (e.g. "here"). Peirce called them *indices*. Less discussed today is Peirce's third kind of sign. *Icons* are signs which resemble what they signify. These are not symbols, as resemblances need not be established by convention. Examples include, crucially, diagrams which function by mimicking the structures they signify – whose parts (as Peirce puts it) bear the same relationship to one another as the parts of the object they represent.

Drawing on this distinction, I will make two key claims. Firstly, **ARG)** considered as a whole serves as an icon.[9] In some very general sense the juxtaposition of those three propositions has an internal structure by appreciating which we see that if **P1** and **P2** are true, **C** *must* be. Necessary reasoning is in essence just a recognition that a certain structure has the particular structure it has, and formal logic is famously structural. Secondly, an icon, or 'logical diagram', and its object *share partial identity*. This is not true for indices and symbols. Since indices serve as a pure pointer, whatever internal properties they might have are irrelevant to their signification, and symbols have nothing in common with their objects by definition since their establishing convention is arbitrary. But as icons represent their objects by resembling them, the basis of that resemblance must be some shared property. Thus Peirce wrote: "…a pure icon does not draw any distinction between itself and its object… whatever it is like, *it in so far is*."[10]

Metaphysically speaking, insofar as one follows **ARG),** written sign and act of inference partake in a larger structure: logical form. Thus what explains the intriguing bindingness between sign and inference which Achilles grasps but cannot explain to the Tortoise is *identity*. What else could explain it? Thus in an important sense the Tortoise fails to read **ARG)** at all.

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[7] Peirce worked and reworked these key definitions throughout his life, but see in particular his *Collected Papers*, ed. C. Hartshorne and P. Weiss (Cambridge, Mass.: Harvard University Press, 1931-1958), sections 2.304, 3.363, 3.641 and 5.74.

[8] John Perry, "The Problem of the Essential Indexical", *Noûs* **13** (1979), pp. 3–21.

[9] This is not to say that it does not have indexical and symbolic aspects as well – the three sign types are functional rather than mutually exclusive. Unfortunately space does not permit me to explain this further here.

[10] Peirce, *Collected Papers*, 5.74 – my emphasis.

Showing Logical Form.

Logical form may be recorded in signs such as **ARG)** and presented to the human mind so that it can be understood and acted upon. And what more could we ask in order to say that a system of signs represents something, or has genuine content? Yet **ARG)** does not *state* logical form in any sense fitting 20th century representationalism. *Pace* the logical positivists, it does not offer an empirical hypothesis. *Pace* Quine, when **P1**, **P2** and **C** are assembled, the structure does not gain its validity by denoting further objects, over and above those denoted by the individual propositions. What are the implications of this expressivism for *realism about logic*? For instance, does it show that logic does not 'talk about real mind-independent things'? No, but I shall argue our notion of 'real mind-independent things' requires some surgery.

It was noted that much analytic philosophy is semantics-driven. Metaphysical realists sometimes rightly express discomfort about this, as it would seem their very realism should lead them to separate semantics and metaphysics. I claim: actually it is fine to derive one's metaphysics from one's semantics – just please get a less simplistic semantics! We may understand Quine's criterion of ontological commitment in Peircean terms as an attempt to place the full burden of representing reality onto *indexical signs*. This leads philosophers with realist sympathies to ask questions such as: "Does term X [e.g. "the Good", "God"…] denote an existent object?", and it often seems hard to answer "yes" for key terms in manifestly important human discourses. On the other hand, we have seen those unsatisfied with metaphysical realism's problematization of such areas often counter with a *conventionalism* which argues the term does not denote but has some other socially sanctioned and taught function. We may understand such conventionalism in Peircean terms as trying to understand all signification as *symbolic*.

Metaphysical realism and conventionalism are assumed to be polar opposites. In so many dialectics in so many philosophy papers an argument against metaphysical realism is assumed without question to be an argument for conventionalism, and *vice versa*. But this dichotomy is false. A third kind of signification exists which does not consist in brute denotation *or* arbitrary convention, but presents structure directly to the mind's eye. It is barely glimpsed in formal semantics today. Yet it is this kind of sign that represents *logical form* – hardly a trivial part of our conceptual scheme. If we could only recognize that the symbol, index and icon all have a unique and irreducible semantic role, and that reality correspondingly comprises real habits, real particulars *and* real structures, we could take an unanticipated leap towards understanding this most contested term.