have shown that motions across the plane of the fan lead to quasi-cylindrical current structures aligned to the spine, in other words, to spine current reconnection. By the same token, advection across the spine is associated with a global current sheet aligned to the fan.

Both spine and fan formulations naturally reproduce the analytic, two-dimensional, shear flow solutions of Craig & Henton (1995). A key feature of the two-dimensional solution is a breakdown of the strict X-point symmetry traditionally associated with planar reconnection models. Only one separatrix plane has flow across it; the other is aligned to a global current sheet (of width $\eta^{1/2}$) across which there is no flow. Fast reconnection is maintained by the pileup of flux at the onset of the sheet.

Conversely, the morphology of the three-dimensional solution can be anticipated from the planar analysis. Whether spine or fan reconnection is achieved depends upon which of the two-dimensional separatrix planes collapses into the spine. This is governed, in the present analysis, by the form of the nonlinear disturbance field $\boldsymbol{Q}$. For instance, in spine reconnection, the current plane collapses but the advection plane is preserved as the fan. What remains are tubes of current aligned to the spine axis, and it is the distribution of current over the spine that determines whether reconnection occurs. Thus, in the axisymmetric case, it is only the $m = 1$ azimuthal mode that allows reconnection, all other modes having vanishing currents at the neutral point. By contrast, a much simpler current structure is associated with fan reconnection, namely, a one-dimensional current sheet overlying the neutral point.

In summary, for isolated X-points we have argued that spine and fan reconnection provide the archetypal forms for steady state magnetic merging in three dimensions. Of course, in more general circumstances—for example, in dynamic compressible plasmas—we would not expect "pure" spine or fan reconnection to occur. In such cases, it seems likely that hybrid forms of reconnection may evolve in which localized currents are distributed over all the separatrices of the field.

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APPENDIX

THE CYLINDRICAL SPINE EQUATION

A1. BOUNDARY LAYER ANALYSIS

Suppose we consider equation (4.8) on the assumption that the resistive term

$$\eta\left(f'' + \frac{f'}{r} - \frac{m^2}{r^2} f\right) = 0 \tag{A1}$$

determines an inner solution, valid for small r. Solutions which remain finite at the origin are given by

$$f_I(r) = r^m . \tag{A2}$$

The outer solution $f_0(r) = A/r^2$ (eq. [4.6]) is obtained by solving

$$f + \tfrac{1}{2} r f' = 0 . \tag{A3}$$

This is just the spine equation in the case $\eta = 0$.

The inner solution (A2) implies that equation (A3) has a contribution of order r^m as $r \to 0$. The fact that equation (A3) maintains a finite contribution for $m = 0$ implies that the inner and outer approximations break down for this mode.

A2. SERIES SOLUTIONS

If we write the spine equation (4.8) in the form

$$x^2 f'' + (x + \tfrac{1}{2} x^3) f' + (x^2 - m^2) f = 0 , \tag{A4}$$

where

$$f = f(x) , \quad x = \left[\frac{\eta}{\alpha(\lambda^2 - 1)}\right]^{-1/2} r ,$$

and assume the series expansion

$$f(x) = x^\nu \sum_{l=0}^{\infty} a_l x^l , \tag{A5}$$

we obtain the two-term recurrence relation

$$a_{l+2} = a_l \frac{1 + (1/2)(\nu + l)}{m^2 - (\nu + l + 2)^2} , \qquad \nu = \pm m , \tag{A6}$$

where $a_0 = 1$, $a_1 = 0$. In the case $\nu = -m$, the series terminates for even m. Given these particular integrals, $p(x)$ say, we can deduce general solutions, for example,

$$p(x) = \frac{1}{x^2}, \qquad f(x) = \frac{1}{x^2}\,[a + b(x^2 + 4)e^{-x^2/4}], \qquad m = 2. \tag{A7}$$

The unknown coefficients must be chosen to ensure the good behavior of the solution close to the origin (i.e., $b = -a/4$). Similarly,

$$p(x) = \frac{1}{x^4} - \frac{1}{12x^2}, \qquad f(x) = \frac{1}{x^4}\,[a(x^2 - 12) + b(x^4 + 16x^2 + 96)e^{-x^2/4}], \qquad m = 4, \tag{A8}$$

with $b = a/8$. Obviously, we can obtain general solutions of this form for all even m.

A3. THE RECONNECTION SOLUTION

The case of most interest physically is $m = 1$. This is the only mode which allows finite current at the neutral point, and hence topological reconnection. Although it is not possible to obtain closed form solutions for odd m, we can use the series expansion (A5) as an inner approximation to match the outer solution $f_0(r) = A/r^2$. The result of matching the outer solution to an eight-term inner expansion is shown in Figure 2a (see § 4.3).

Finally, we mention the scaling of the reconnection rate with resistivity. If we normalize the outer field so that $f_0(1) = 1$, then the inner solution $f_I(r) = ar + O(r^3)$ must achieve the amplitude $1/r^2$ when $r^2 \simeq \eta$. Thus, a must scale as $\eta^{-3/2}$ to maintain a fixed field amplitude on the outer boundary. Accordingly, the current density at the neutral point must build up as $\eta^{-3/2}$. This implies that both the ohmic dissipation rate and the flux annihilation rate achieve the superfast scaling η^{-1} and $\eta^{-1/2}$.

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